STRANGE VECTOR FORM FACTOR OF KAONS

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Abstract

Starting from the $\omega - \phi$ mixing, further assuming the coupling of the quark-current of some flavour to be of universal strenth exclusively to the component of vector-meson wave function with the same flavour and finally taking numerical values of the coupling constant ratios $(f_{\omega K\bar{K}}/f^e_\omega)$, $(f_{\phi K\bar{K}}/f^e_\phi)$ from the isoscalar part of a realistic six-resonance unitary and analytic model of the kaon electromagnetic structure, the strange-quark vector current form factor behaviour of K-mesons in space-like and time-like regions is predicted.

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Indirect experimental evidence for strangeness in nucleon in the determination of the πN σ-term, in polarized deep-inelastic lepton-nucleon scattering, in violations of the OZI rule, in elastic νp scattering and in neutrino charm production motivated in recent years, among others, also a realization of well-defined experimental [1-5] and theoretical [6-30] program of an investigation of the matrix element $\langle N|\bar{s}\gamma_\mu s|N\rangle$, which is directly related to the strange nucleon electric and magnetic form factors (ff's), G^s_E and G^s_M. Although the present experimental values [1-5] of the strange nucleon ff's are almost consistent with zero, the rather sizeable error bars document the difficulty of such type experiments and no definitive conclusion can as yet be made regarding the experimental scale of $\langle N|\bar{s}\gamma_\mu s|N\rangle$. On the other hand the theoretical understanding of $\langle N|\bar{s}\gamma_\mu s|N\rangle$ is even much less clear. There have been many theoretical attempts to predict the nucleon strange form factors, differing by the methods used to study the non-perturbative physics behind them: VMD models with $\omega - \phi$ mixing [6,10,15,19], Skyrme models [7,9], Nambu-Jona-Lasinio soliton model [14], SU(3) chiral bag model [20], kaon loop model [8,11,13,17], meson exchange model [16], lattice QCD [21,26,27,28], chiral quark model [25], SU(3) chiral quark-soliton model [30], HBChPT [22,24], unitary and analytic model with $\omega - \phi$ mixing [29] and dispersion relation approach [23]. So, the range of predictions for the strange nucleon ff's is broad and obtained results are mutulally contradicting. The latter illustrates the sensitivity of see quark observables to model assumptions and, on the other hand, the limited usefulness of models in making realistic predictions. One could hope for more insight from ChPT, which relies on the chiral symmetry of QCD. However, the application of ChPT is restricted in energy and mereover, depends [24] on unknown counterterms, which have to be determined from data, including the data on the strange nucleon ff's to be charged by sizable error bars, for which one expects predictions just from ChPT. In this paper we turn our attention to the dispersion relation approach [23], which, similarly to ChPT, relies on general principles of quantum field theory, including also QCD, and seems to bring some insight into $\langle N|\bar{s}\gamma_\mu s|N\rangle$ as well, relating existing experimental data to the observables of interest. In the dispersion relation approach it is analyticity and unitarity, rather than chiral symmetry, which allow one to make such a relation. Practically it means first, on the base of the assumed analytic properties and the asymptotic behaviours of ff's, to write down substracted (or unsubracted) dispersion integral relations and then to use the unitarity condition for a derivation of relations of the imaginary parts of ff's under the integrals to amplitudes of physical processes. Though, there is an infinit number of contributing terms into imaginary parts of ff's, the lightest intermediate states in the unitarity condition generate dominant contributions to the leading moments of the strange current. The lowest one is the 3π contribution, which can resonate into a state having the same quantum numbers as the ϕ-meson (nearly pure $s\bar{s}$ state). Thus, the 3π state can contribute appreciably to the strange nucleon ff's via its coupling to the ϕ-meson. The next important is the $K\bar{K}$ intermediate state, which is the lightest state containing valence strange quarks and represents so-called "kaon cloud dominance" in the strange nucleon ff's.

In order to estimate $K\bar{K}$ contributions to the corresponding spectral functions, following the analysis in [31], one can express e.g. the absorptive part of the strange nucleon electric ff $G_E^s(t)$ (and similarly for $G_M^s(t)$) as a product of the appropriate J=1 $K\bar{K} \to N\bar{N}$ partial wave amplitude $b_1^{\lambda,\lambda'}$ and the strange vector form factor of kaons $F_K^s(t)$ as follows [32]

$$ImG_E^s = Re\left\{\left(\frac{q}{4m_N}\right) b_1^{1/2,1/2}(t) \cdot F_K^s(t)^*\right\}, \tag{1}$$

where $q = \sqrt{t/4 - m_K^2}$ and λ, λ' in $b_1^{\lambda,\lambda'}$ are denoting corresponding helicities.

In the spirit of a discussion above, that futher theoretical improvements are needed to constrain the uncertainties in the experimental measurements of the strange nucleon ff's, the problem is now to determine $b_1^{\lambda,\lambda'}$ and $F_K^s(t)$ as reliably as possible.

Whereas the $K\bar{K} \to N\bar{N}$ J=1 partial wave $b_1^{1/2,1/2}$ can be determined [31,32] by an analytic continuation of the data on the $K^+N \to K^+N$ amplitude into the unphysical region, there exists no experimental infomation on $F_K^s(t)$ and one has to find a position in some specific models of $F_K^s(t)$, like a simple ϕ-dominance form [31], Gounaris-Sakurai parametrization [33] of the electromagnetic (EM) ff $F_\pi^{EM}(t)$ with the replacement of the ρ mass and width with those of ϕ, or the sum of Breit-Wigner terms [23] of the form [34]

$$F_K^s = \sum_{v=\omega,\phi} C_v^s \frac{m_v^2}{m_v^2 - t - im_v\Gamma_v f_v(t)} \tag{2}$$

where $f_v(t) = t/m_v^2$ and C_v^s are determined by the relations [6]

$$\begin{aligned} C_\omega^s/C_\omega^e &\sim -0.2 \\ C_\phi^s/C_\phi^e &\sim -3 \end{aligned} \tag{3}$$

from the values of C_ω^e, C_ϕ^e determined in a fit of data on the kaon EM ff's measured by $e^+e^- \to K\bar{K}$ processes.

In this paper we follow the procedure in [23], but for a parametrization of the kaon EM ff's and kaon strange ff we apply more sophisticated unitary and analytic models, which unify all known ff properties always into one analytic function in a very natural way and as a result, one can expect to obtaine realistic behaviour of $F_K^s(t)$.

The strange-quark vector current ff of K-mesons $F_K^s(t)$ is defined by the matrix element of the strange-quark current $J_\mu^s = \bar{s}\gamma_\mu s$

$$\langle K(k')|\bar{\mathrm{s}}\gamma_\mu \mathrm{s}|K(k)\rangle = (k + k^{'})_\mu F_K^s(t) \tag{4}$$

in analogy to a definition of the EM ff's of K -mesons, $F_{K^+}(t)$ and $F_{K^0}(t)$

$$\langle K(k')|J_\mu^{EM}|K(k)\rangle = (k + k')_\mu F_K(t) \tag{5}$$

where

$$J_\mu^{\mathrm{EM}} = \frac{2}{3}\bar{\mathrm{u}}\gamma_\mu \mathrm{u} - \frac{1}{3}\bar{\mathrm{d}}\gamma_\mu \mathrm{d} - \frac{1}{3}\bar{\mathrm{s}}\gamma_\mu \mathrm{s} \tag{6}$$

is the EM current operator written by means of the u, d and s quark fields, k and k' are four momenta of K-mesons and $t = (k' - k)^2 = q^2 = -Q^2$ is the four momentum transfer squared.

The ff's $F_{K^+}(t)$ and $F_{K^0}(t)$ can be decomposed into isoscalar and isovector parts

$$\begin{aligned} F_{K^+} &= F_K^{I=0}(t) + F_K^{I=1}(t) \\ F_{K^0} &= F_K^{I=0}(t) - F_K^{I=1}(t) \end{aligned} \tag{7}$$

to be defined by the following matrix elements

$$\begin{aligned} \langle K(k')|J_\mu^{I=0}|K(k)\rangle &= (k + k')_\mu F_K^{I=0}(t) \\ \langle K(k')|J_\mu^{I=1}|K(k)\rangle &= (k + k')_\mu F_K^{I=1}(t), \end{aligned} \tag{8}$$

where $J_\mu^{I=0} = \frac{1}{6}(\bar{\mathrm{u}}\gamma_\mu u + \bar{\mathrm{d}}\gamma_\mu \mathrm{d}) - \frac{1}{3}\bar{\mathrm{s}}\gamma_\mu \mathrm{s}$ and $J_\mu^{I=1} = \frac{1}{2}(\bar{\mathrm{u}}\gamma_\mu u - \bar{\mathrm{d}}\gamma_\mu \mathrm{d})$ are isoscalar and isovector parts of the EM current J_μ^{EM} given by (6), respectively.

Since the strange-quark vector current $J_\mu^s = \bar{\mathrm{s}}\gamma_\mu \mathrm{s}$ carries the quantum numbers of the isoscalar part of the EM current $J_\mu^{I=0}$, then the strange ff $F_K^s(t)$ can contribute only to a behaviour of the isoscalar part $F_K^{I=0}(t)$ of the kaon EM ff's. So, it is natural to expect that in principle one could draw out the behaviour of $F_K^s(t)$ just from the isoscalar part $F_K^{I=0}(t)$ of the kaon EM ff's. Really, if there is a suitable model of

$$F_K^{I=0}(t) = f[t; (f_{\omega KK}/f_\omega^e), (f_{\phi KK}/f_\phi^e)] \tag{9}$$

and

$$F_K^s(t) = \tilde{f}[t; (f_{\omega KK}/f_\omega^s), (f_{\phi KK}/f_\phi^s)] \tag{10}$$

is the model of the same analytic structure, but with different norm and different asymptotic behaviour (therefore denoted by $\tilde{f}$), then starting from the ω-ϕ mixing and assuming that the quark-current of some flavour couples with universal strength exclusively to the component of the vector-meson wave function with the same flavour, one can prove [6] the following relations

$$\begin{aligned}
(f_{\omega KK}/f_\omega^s) &= -\sqrt{6}\frac{\sin\epsilon}{\sin(\epsilon+\theta_0)}(f_{\omega KK}/f_\omega^e); \qquad (11)\\
(f_{\phi KK}/f_\phi^s) &= -\sqrt{6}\frac{\cos(\epsilon)}{\cos(\epsilon+\theta_0)}(f_{\phi KK}/f_\phi^e)
\end{aligned}$$

between parameters of the models (10) and (9), where $\epsilon = 3.7^\circ$ is a deviation from the ideally mixing angle $\theta_0 = 35.3^\circ$, and as a result the behaviour of $F_K^s(t)$ can be predicted.

Practically the EM structure of K-mesons we describe by the unitary and analytic model [35,36]

$$\begin{aligned}
F_K^{I=0}[V(t)] &= \left(\frac{1-V^2}{1-V_N^2}\right)^2 \left[\frac{1}{2}\frac{(V_N-V_{\phi'})(V_N-V_{\phi'}^*)(V_N+V_{\phi'})(V_N+V_{\phi'}^*)}{(V-V_{\phi'})(V-V_{\phi'}^*)(V+V_{\phi'})(V+V_{\phi'}^*)} + \right. \qquad (12)\\
&+ \left\{\frac{(V_N-V_\omega)(V_N-V_\omega^*)(V_N-1/V_\omega)(V_N-1/V_\omega^*)}{(V-V_\omega)(V-V_\omega^*)(V-1/V_\omega)(V-1/V_\omega^*)} - \right.\\
&- \left.\frac{(V_N-V_{\phi'})(V_N-V_{\phi'}^*)(V_N+V_{\phi'})(V_N+V_{\phi'}^*)}{(V-V_{\phi'})(V-V_{\phi'}^*)(V+V_{\phi'})(V+V_{\phi'}^*)}\right\}(f_{\omega KK}/f_\omega^e) +\\
&+ \left\{\frac{(V_N-V_\phi)(V_N-V_\phi^*)(V_N-1/V_\phi)(V_N-1/V_\phi^*)}{(V-V_\phi)(V-V_\phi^*)(V-1/V_\phi)(V-1/V_\phi^*)} - \right.\\
&- \left.\left.\frac{(V_N-V_{\phi'})(V_N-V_{\phi'}^*)(V_N+V_{\phi'})(V_N+V_{\phi'}^*)}{(V-V_{\phi'})(V-V_{\phi'}^*)(V+V_{\phi'})(V+V_{\phi'}^*)}\right\}(f_{\phi KK}/f_\phi^e)\right]
\end{aligned}$$

$$\begin{aligned}
F_K^{I=1}[W(t)] &= \left(\frac{1-W^2}{1-W_N^2}\right)^2 \left[\frac{1}{2}\frac{(W_N-W_{\varrho'''})(W_N-W_{\varrho'''}^*)(W_N+W_{\varrho'''})(W_N+W_{\varrho'''}^*)}{(W-W_{\varrho'''})(W-W_{\varrho'''}^*)(W+W_{\varrho'''})(W+W_{\varrho'''}^*)} + \right.\\
&+ \left\{\frac{(W_N-W_\varrho)(W_N-W_\varrho^*)(W_N-1/W_\varrho)(W_N-1/W_\varrho^*)}{(W-W_\varrho)(W-W_\varrho^*)(W-1/W_\varrho)(W-1/W_\varrho^*)} - \right. \qquad (13)
\end{aligned}$$

$$
\begin{aligned}
&- \left.\frac{(W_N-W_{\varrho'''})(W_N-W^*_{\varrho'''})(W_N+W_{\varrho'''})(W_N+W^*_{\varrho'''})}{(W-W_{\varrho'''})(W-W^*_{\varrho'''})(W+W_{\varrho'''})(W+W^*_{\varrho'''})}\right\}(f_{\varrho KK}/f^e_{\varrho}) + \\
&+ \left\{\frac{(W_N-W_{\varrho'})(W_N-W^*_{\varrho'})(W_N-1/W_{\varrho'})(W_N-1/W^*_{\varrho'})}{(W-W_{\varrho'})(W-W^*_{\varrho'})(W-1/W_{\varrho'})(W-1/W^*_{\varrho'})} - \right.\\
&- \left.\left.\frac{(W_N-W_{\varrho'''})(W_N-W^*_{\varrho'''})(W_N+W_{\varrho'''})(W_N+W^*_{\varrho'''})}{(W-W_{\varrho'''})(W-W^*_{\varrho'''})(W+W_{\varrho'''})(W+W^*_{\varrho'''})}\right\}(f_{\varrho' KK}/f^e_{\varrho'})\right],
\end{aligned}
$$

where

$$V(t) = i\frac{\sqrt{q_{in}^{I=0}+q}-\sqrt{q_{in}^{I=0}-q}}{\sqrt{q_{in}^{I=0}+q}+\sqrt{q_{in}^{I=0}-q}}$$

$$q = [(t-t_0^{I=0})/t_0^{I=0}]^{1/2}; \quad q_{in} = [(t_{in}-t_0^{I=0})/t_0^{I=0}]^{1/2}; \quad V_N = V(t)_{|t=0};$$

$t_0^{I=0} = 9m_\pi^2$, $t_{in}^{I=0}$ is an effective square-root branch point simulating contributions of all higher thresholds given by the unitarity conditions, V_i $(i=\omega,\phi,\phi')$ are the positions of vector-meson poles in $V(t)$-plane and similarly for $W(t)$, W_N and W_j $(j=\varrho,\varrho',\varrho''')$.

The ff's $F_K^{I=0}[V(t)]$ and $F_K^{I=1}[W(t)]$ reflect all known theoretical properties of the kaon EM ff's.

Similarly, we construct the unitary and analytic model of the strange-quark vector current ff of K-mesons

$$
\begin{aligned}
F_K^s[V(t)] &= \left(\frac{1-V^2}{1-V_N^2}\right)^6\left[-\frac{(V_N-V_{\phi'})(V_N-V^*_{\phi'})(V_N+V_{\phi'})(V_N+V^*_{\phi'})}{(V-V_{\phi'})(V-V^*_{\phi'})(V+V_{\phi'})(V+V^*_{\phi'})} + \right. \qquad (14)\\
&+ \left\{\frac{(V_N-V_{\omega})(V_N-V^*_{\omega})(V_N-1/V_{\omega})(V_N-1/V^*_{\omega})}{(V-V_{\omega})(V-V^*_{\omega})(V-1/V_{\omega})(V-1/V^*_{\omega})} - \right.\\
&- \left.\frac{(V_N-V_{\phi'})(V_N-V^*_{\phi'})(V_N+V_{\phi'})(V_N+V^*_{\phi'})}{(V-V_{\phi'})(V-V^*_{\phi'})(V+V_{\phi'})(V+V^*_{\phi'})}\right\}(f_{\omega KK}/f^s_{\omega}) + \\
&+ \left\{\frac{(V_N-V_{\phi})(V_N-V^*_{\phi})(V_N-1/V_{\phi})(V_N-1/V^*_{\phi})}{(V-V_{\phi})(V-V^*_{\phi})(V-1/V_{\phi})(V-1/V^*_{\phi})} - \right.\\
&- \left.\left.\frac{(V_N-V_{\phi'})(V_N-V^*_{\phi'})(V_N+V_{\phi'})(V_N+V^*_{\phi'})}{(V-V_{\phi'})(V-V^*_{\phi'})(V+V_{\phi'})(V+V^*_{\phi'})}\right\}(f_{\phi KK}/f^s_{\phi})\right]
\end{aligned}
$$

with the same analytic structure as $F_K^{I=0}[V(t)]$, but to be normalized to -1, in order to take the correct strangeness charge into account, and with the asymptotic behaviour

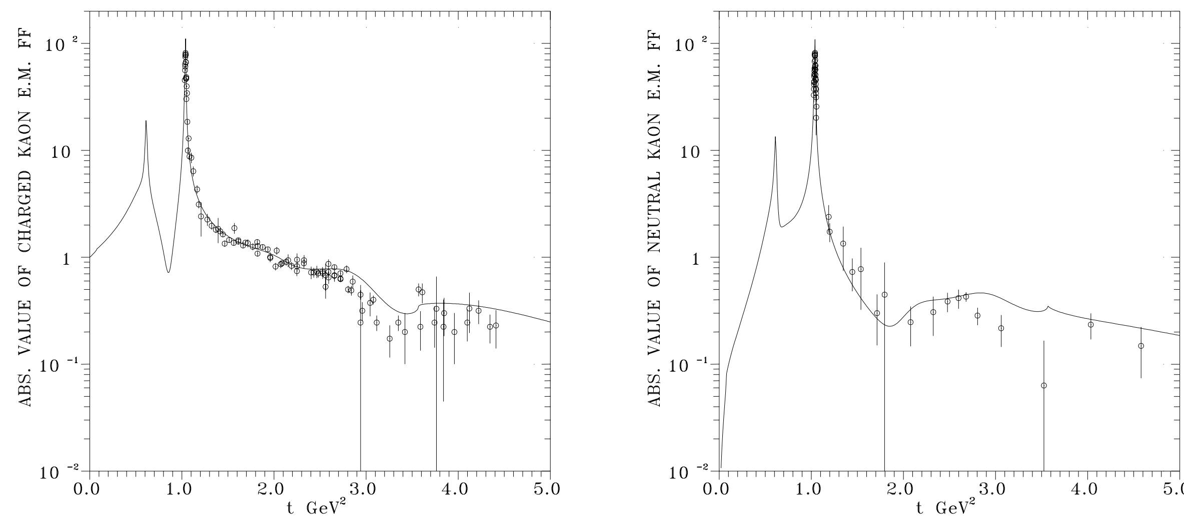

Figure 1: Description of data on the charge and neutral kaon EM ff's by unitary and analytic models.

$F^s_K(t)_{|t|\to\infty} \sim t^{-3}$, as in addition to the valence $(q\bar{q})$ of K-mesons, now also the $s\bar{s}$ - pairs contribute and as a result there is totally 4 quarks inside the kaons. The f^s_ω and f^s_ϕ are strangeness-current-vector-meson (ω and ϕ, respectively) coupling constants.

With the aim of a determination of free parameters in isoscalar and isovector parts of EM ff's of K- mesons, $F^{I=0}_K[V(t)]$ and $F^{I=1}_K[W(t)]$, the latter are compared through the relations (7) with all existing data on F_{K^+} and F_{K^0}, simultaneously. The results with χ^2/ndf=1.6 are graphically presented in Fig.1. Important parameters of $F^{I=0}_K[V(t)]$ for a prediction of $F^s_K[V(t)]$ behaviour are found to be

$$t^{I=0}_{in} \equiv t^s_{in} = 1.0480 \pm 0.2800 \mathrm{GeV}^2 \tag{15}$$

$$(f_{\omega KK}/f^e_\omega) = 0.2076 \pm 0.0542; \quad (f_{\phi KK}/f^e_\phi) = 0.3466 \pm 0.0491$$

from where, by means of the relations (11), one finally evaluates unknown parameters of $F^s_K[V(t)]$ in (10) to be

$$(f_{\omega KK}/f^s_\omega) = -0.0522; \quad (f_{\phi KK}/f^s_\phi) = -1.0902. \tag{16}$$

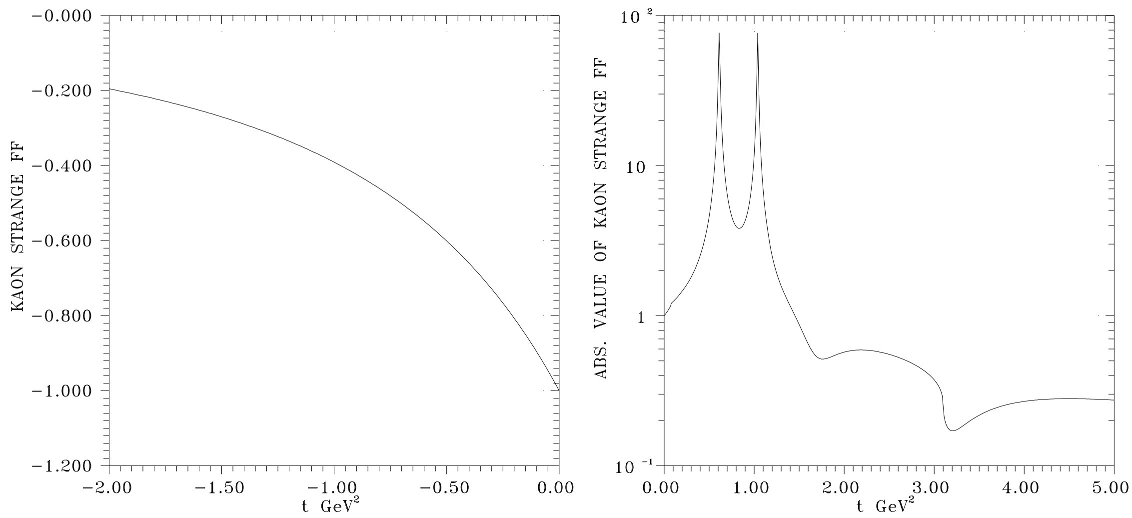

Figure 2: Prediction of the strange-quark vector current ff of K-mesons in the space-like and the time-like regions.

Then by means of the unitary and analytic model (14) the strange-quark vector current form factor behaviour of K-mesons in space-like and time-like regions, as graphically presented in Fig.2, is predicted which is formed also by ϕ-meson, similarly to [23], but besides the latter also ω-meson contributes and the height and the shape of $F_K^s(t)$ are different.

A prediction of a corresponding spectral function behavior is shown in Fig.3. Here we would like to note, that the obtained results depend on the precision of existing data on the kaon EM ff's. One hopes that the substantial improvement, at least at the ϕ-region, will be obtained by VEPP-2M and DAΦNE soon. Consequently even more accurate predictions for $F_K^s[V(t)]$ behaviour could be achieved.

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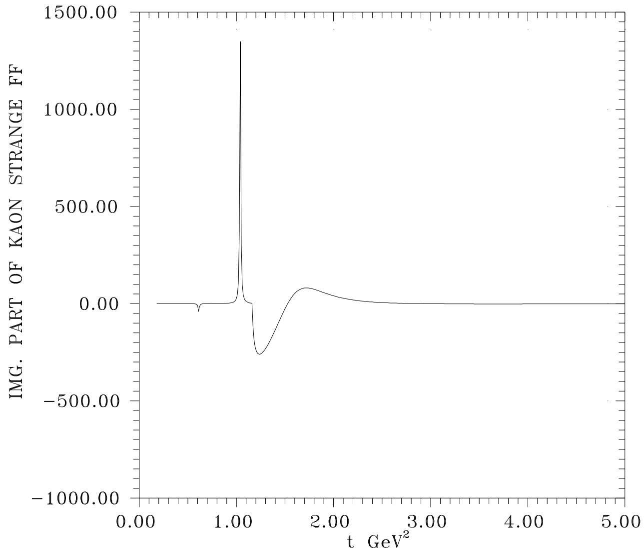

Figure 3: The predicted spectral function behaviour of $F_K^s(t)$.

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